BOUNDARY LAYER WITH SELECTIVE SUCTION

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The heat- and mass-transfer processes in a boundary layer with porous suction of one of the components of the gas mixture are analyzed. It is shown that for selective suction there is a limiting penetration parameter, and the asymptotic suction regime is not attained.

In various branches of engineering it is necessary to deal with processes of mass suction from a boundary layer. For example, boundary-layer suction is used to lower the drag on aircraft by increasing the critical Reynolds number for the laminar-to-turbulent flow transition. Boundary-layer suction is an effective tool for the intensification of heat- and mass-transfer processes. The transverse flow of material toward the wall is also typical of such processes as condensation, adsorption, chemical reactions with wall deposition of the reaction products, etc.

All of the above-indicated processes, despite the dissimilarity in the physics of the effects, share common laws governing the transfer of momentum, energy, and mass in the surface layer, a fact that has enabled a number of authors [1-3] to solve complex problems in the condensation of vapor from vapor–gas mixtures, using the methods of boundary-layer theory. This approach has also been found extremely useful in generalizing experimental data on vapor condensation at cryogenic temperatures [4]. The simple analytical relations derived in these studies on the basis of the asymptotic theory of turbulent boundary layers [5] have enabled the authors to analyze, within practical engineering error limits, the influence of various factors on heat and mass transfer in condensation, specifically: the intensity of the transverse flow of material, the ratio of the molecular weights of the condensing and inert components, etc. In this case the influence of the formed condensate film on the heat and mass transfer has been disregarded, along with the dynamic interaction of the layers at the phase interface. In essence, the problem has been reduced to solving the boundary-layer equations on a plate with porous suction.

In the present article we give the results of a theoretical analysis of a turbulent boundary layer with selective suction, where only one component is extracted from the flowing gas mixture through the wall. This suction regime is feasible, for example, in the condensation of vapor from a vapor–gas mixture, where the molecular weights of the uncondensed gas and the vapor transported through the wall can differ, as well as in boundary layers with chemical reactions on a surface, where only the oxidant diffuses from the gas mixture in the flow core toward the wall.

Investigations of the process associated with vapor condensation from vapor–gas mixtures have been reported in a vast number of experimental and theoretical papers [1-4, 6-8]. In some papers [3, 4] it is proposed that the analytical relations derived for blowing, i.e., porous injection of a foreign substance, be used with an appropriate change of sign for the permeability parameter to determine the heat- and mass-transfer function. The most detailed study of the process of condensation of inhomogeneous vapor–gas mixtures in turbulent flow is reported in [2], but inaccuracies in the statement of the problem, which are discussed below, make these results applicable only in a special case.

We consider the process of flow around a plate in the general case of a binary nonisothermal mixture of gases with nonuniform suction. The integral diffusion relation written for the i-th component of the gas mixture in the given boundary layer has the form [5]

$$\frac{d\,\mathrm{Re}_d^{**}}{d\bar{x}} + \frac{\mathrm{Re}_d^{**}}{\Delta\tilde{k}_i}\frac{d\Delta\tilde{k}_i}{d\bar{x}} = \mathrm{Re}_L \Psi \mathrm{St}_0 (1 - b_{1d}), \quad (1)$$

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where $Re_d^{**} = \rho_0 w_0 \delta_d^{**}/\mu$ is the Reynolds number formed with respect to the mass-reduction thickness;

$$\delta_d^{**} = \int_0^\delta \frac{\rho w}{\rho_0 w_0}\left[1 - \frac{\tilde{k}_i - (\tilde{k}_i)_w}{(\tilde{k}_i)_0 - (\tilde{k}_i)_w}\right] dy;$$

and $\Delta\tilde{k}_i = (\tilde{k}_i)_0 - (\tilde{k}_i)_w$. The penetration parameter b_{1d}, which characterizes the suction rate, is given by the relation $b_{1d} = j_w/\rho_0 w_0 St_d$, and the diffusion Stanton number for the investigated conditions is equal to

$$St_d = \frac{-\rho D\left(\frac{\partial \tilde{k}_i}{\partial y}\right)_w}{\rho_0 w_0 \Delta\tilde{k}_i}. \tag{2}$$

Making use of the similarity of the heat- and mass-transfer processes and friction, we find the concentration distribution of the suction component (which we designate hereinafter by the index 1) in the thickness of the boundary layer:

$$\tilde{k}_1 = \tilde{k}_{1w}(1-\omega) + \tilde{k}_{10}\omega, \tag{3}$$

and we determine the wall value of the concentration from the balance of mass on the porous surface:

$$j_w = j_w \tilde{k}_{1w} - \rho D\left(\frac{\partial \tilde{k}_i}{\partial y}\right)_w. \tag{4}$$

Then from (2) and (4) we obtain

$$\tilde{k}_{1w} = \frac{\tilde{k}_{10} - b_{1d}}{1 - b_{1d}}. \tag{5}$$

In Eq. (5) the penetration parameter b_{1d} must be positive in suction.

It is well known that the penetration parameter b_1 for the suction of isothermal [5] and nonisothermal [10] boundary layers can vary in the interval $0 \leqslant b_1 \leqslant 1$. For $b_1 = 1$ the asymptotic suction regime is attained wherein the transfer processes do not depend on the flow regime or the properties of the gas. This regime is determined by the condition $\Psi = b$, where $\Psi = (St/St_0)_{Re^{**}}$, $b = b_1\Psi = j_w/\rho_0 w_0 St_0$, and St_0 is the dimensionless heat-transfer coefficient (relative to the mass transfer, i.e., the Stanton number) under standard conditions.

In selective suction, however, where only one component of the gas mixture can be transported through the wall, it follows from relation (5) that the penetration parameter b_1 has a limiting value and cannot be greater than the concentration of the suction component in the flow core: $(b_1)_{lm} \leqslant \tilde{k}_{10}$. This condition has the important implication that in the given boundary-layer suction it is impossible to attain the asymptotic suction regime $b_1 = 1$, and only in the limiting case when the concentration of the suction component in the flow core tends to unity, $\tilde{k}_{10} \to 1$, is self-similar flow attained. It must be noted that the existence of a limiting suction parameter $(b_1)_{lm}$ and its value are solely attributable to the selectivity of the suction and do not depend on the degree of inhomogeneity of the gas mixture in the flow core, i.e., on the ratio of the molecular weights of the components transported and not transported through the wall. The indicated effect will also occur for gas mixtures containing substances with similar physical properties, for example in the condensation of water vapor in a mixture with air, nitrogen, and oxygen at cryogenic temperatures, etc.

Thus, the concentration of the suction component in the flow core is what determines and limits the intensity of the process of material suction from the boundary layer. The existence of the limiting penetration parameter $(b_1)_{lm}$ also affects the heat-transfer process in this suction regime. As an illustration we write the equation for the conservation of energy at the wall in the condensation of vapor from a vapor–gas mixture, assuming that the energy admitted to the wall is associated entirely with the heat of phase transition:

$$q_w = j_w r \tag{6}$$

or, transforming, we obtain

$$St = \frac{j_w}{\rho_0 w_0} K, \quad \Psi_T = b_T K, \tag{7}$$

where the Stanton number is equal to $St = q_w/\rho_0 w_0(h_w - h_0)$, and the Kutateladze criterion is defined as $K = r/(h_w - h_0)$.

In the condensation of a pure vapor in the asymptotic suction regime, i.e., for $\Psi = b_T$, we infer from (7) that the asymptotic value of the phase-transition criterion is $K_{as} = 1$. In the case of vapor condensation from a mixture with a gas, assuming in the first approximation that $b_1 = b_{1d}$ and making use of the fact that $(b_1)_{lm} = \tilde{k}_{10}$, we find that the limiting value of the Kutateladze criterion is determined by the concentration of the suction component in the flow core:

$$K_{1m} = \frac{1}{\tilde{k}_{10}} . \qquad (8)$$

In deriving Eqs. (4)-(8) we have not made any assumptions about the flow regime in the boundary layer, and so the derived relations for the limiting penetration parameters are valid for both laminar and turbulent boundary layers.

To analyze the heat- and mass-transfer processes on the basis of the integral relation (1) it is necessary to determine the dependence of the function Ψ on the suction parameter b_1. As an example we consider a turbulent boundary layer with the suction of one component of a gas mixture under nonisothermal conditions; such relations can also be obtained for laminar flow using, for example, film theory [9].

According to the asymptotic turbulent boundary-layer theory of Kutateladze and Leont'ev [5], the expression for the combined mass-flow heat-transfer and friction function has the form

$$\Psi = \left[\int_0^1 \left(\tilde{\rho} \frac{\tilde{\tau}_0}{\tilde{\tau}} \right)^{1/2} d\omega \right]^2 . \qquad (9)$$

Here the distribution of the tangential stresses over the cross section of the boundary layer is determined by the transverse flow of material toward the wall:

$$\frac{\tau}{\tilde{\tau}_0} = 1 - b_1\omega. \qquad (10)$$

The quantity $\tilde{\rho} = \rho/\rho_0$ represents the distribution of the density of the gas mixture over the thickness of the boundary layer. This parameter characterizes the inhomogeneity of the composition and the influence of nonisothermicity on the heat- and mass-transfer process.

In accordance with the equation of state of a mixture of ideal gases we have

$$\frac{\rho_0}{\rho} = \frac{M_0 T}{M T_0} . \qquad (11)$$

The molecular weight of a binary gas mixture is

$$\frac{1}{M} = \frac{\tilde{k}_1}{M_1} + \frac{1-\tilde{k}_1}{M_2} ,$$

where the index 2 refers to the nonsuction component. Making use of the concentration distribution of the components (3) in conjunction with expressions (4) and (5) for a mixture of gases with the same valence $M_i C p_i$ = const (the admissibility of such an assumption has been proved in [2]), we find the density profile

$$\frac{\rho_0}{\rho} = \psi_1\psi - (1 - \psi_1\psi)\,\omega. \qquad (12)$$

For isothermal flow $\psi = T_w/T_0 = 1$ we obtain

$$\frac{\rho_0}{\rho} = \psi_1 + (1 - \psi_1)\,\omega. \qquad (13)$$

The quantity $\psi_1\psi$ in (12) represents the ratio of the total enthalpies and can be written in the form

$$\psi_1\psi = \left[1 + \frac{b_1}{1+b_1} \left(\frac{1}{M^*} - 1 \right) \right] .$$

It must be noted that expression (12) is identical with the relation for the density in the boundary layer for the injection of a foreign gas [5]; in the suction case M^* is equal to the ratio of the molecular weight M_1 of the suction component to the molecular weight M_0 of the gas mixture in the flow core:

$$M^* = \frac{M_1}{M_0} = \tilde{k}_{10} + \bar{M}(1 - \tilde{k}_{10}), \tag{14}$$

where $\bar{M} = M_1/M_2$. Then the nonuniformity parameter ψ_1 is equal to

$$\psi_1 = 1 + \frac{b_1}{1+b_1}\left[\frac{1}{\tilde{k}_{10} + \bar{M}(1-\tilde{k}_{10})} - 1\right]. \tag{15}$$

It therefore follows from (15) that the nonuniformity parameter ψ_1 is a function of the suction rate b_1, the ratio of the molecular weights of the suction component and the inert component $\bar{M} = M_1/M_2$, and their concentrations in the flow core.

Figure 1 shows the influence of the penetration parameter b_1 on the nonuniformity parameter ψ_1 for the case of condensation of water vapor in a mixture with helium, $\bar{M} = 4.54$, and Freon, $\bar{M} = 0.15$; the calculations were carried out for various values of the vapor concentration $\tilde{k}_{10}$ in the flow core. It is evident from the data that the nonuniformity parameter ψ_1, like the penetration parameter b_1, has a limiting value given by the relation

$$(\psi_1)_{lm} = 1 - \frac{\tilde{k}_{10}}{1 - \tilde{k}_{10}}\left[\frac{1}{\tilde{k}_{10} + \bar{M}(1 - \tilde{k}_{10})} - 1\right]. \tag{16}$$

In this limiting case the concentration of the suction component at the wall tends to zero as $\tilde{k}_{1w} \to 0$.

Figure 2 shows the results of calculations according to (16) of the limiting parameter $(\psi_1)_{lm}$ as a function of the concentration of condensed water vapor in a flow with helium (curve 1), air (curve 2), and Freon (curve 3). It is evident from the figure that the vapor concentration strongly influences the limiting value of the nonuniformity parameter; in the limit where the concentration of the suction component is close to zero, $\tilde{k}_{10} \to 0$, we have a quasiisothermal suction regime $(\psi_1)_{lm} \to 1$, and in the opposite limit $\tilde{k}_{10} \to 1$ we have nonisothermal suction with $(\psi_1)_{lm} \to \bar{M}$. For a homogeneous boundary layer ($\bar{M} = 1$) it follows from (16) that $(\psi_1)_{lm} \equiv 1$ for any values of $\tilde{k}_{10}$. We have subsequently used the data of Fig. 2 to determine the limiting values of the relative mass-flow heat-transfer coefficients (relative Stanton numbers).

Using (10) and (12), we find a solution of the integral (9):

$$(\psi_1)_{lm} > \psi_1\psi > 1,$$

$$\Psi = \frac{4}{b_1(1 - \psi\psi_1)}\left[\ln\frac{\sqrt{(1-\psi_1\psi)(1+b_1)} + \sqrt{b_1}}{\sqrt{1-\psi_1\psi} + \sqrt{\psi_1\psi b_1}}\right]^2, \tag{17}$$

$$(\psi_1)_{lm} < \psi_1\psi < 1,$$

$$\Psi = \frac{4}{b_1(\psi_1\psi - 1)}\left[\operatorname{arctg}\sqrt{\frac{b_1}{(\psi_1\psi - 1)(1+b_1)}} - \operatorname{arctg}\sqrt{\frac{b_1\psi_1\psi}{\psi_1\psi - 1}}\right]^2. \tag{18}$$

Expressions (17) and (18) go over in the uniform-suction case $\psi_1 = 1$ to the equations for the relative Stanton numbers for boundary-layer suction under nonisothermal conditions [10].

It is important to note that a similar analysis of heat- and mass-transfer processes in a turbulent boundary layer with condensation of vapor–gas mixtures has been carried out in [2]. The relative Stanton number Ψ was calculated for a constant concentration of the gas component of the mixture at the wall, and the wall intensity b_1 of the transverse mass flow was specified as a parameter. In this situation, according to expression (5), for the data given in [2] the concentration of the components in the flow core should vary as a function of the penetration parameter b_1, contrary to the true pattern of the process. For example, using data calculated for $\tilde{k}_{1w} = 0$, we obtain for the vapor concentration in the flow core $\tilde{k}_{10} = b_1$, which, as shown above, corresponds to the limiting state of the boundary layer.

Figure 3 shows the relative Stanton number as a function of the penetration parameter $b = j_w/\rho_0 w_0 St_0$ for the selective suction of a quasihomogeneous isothermal turbulent boundary

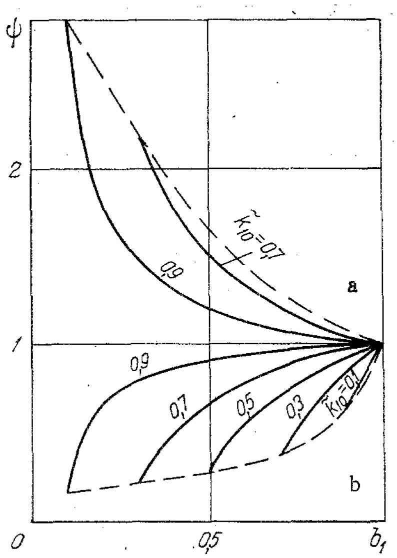

Fig. 1

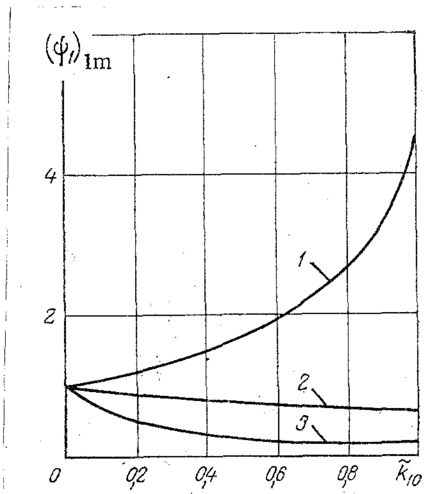

Fig. 2

Fig. 1. Nonuniformity parameter ψ_1 versus suction parameter b_1. a) Water vapor + helium, $\bar{M}$ = 4.54; b) water vapor + Freon-12, $\bar{M}$ = 0.15.

Fig. 2. Influence of inhomogeneity of the boundary layer on the limiting parameter $(\psi_1)_{lm}$. 1) $\bar{M}$ = 4.54; 2) 0.62; 3) 0.15.

layer. Curve 1 in this figure corresponds to a heat-transfer law such that the composition of the material transported through the wall is the same as in the flow core [5]:

$$\Psi = \left(1 + \frac{b}{b_{as}}\right)^2, \tag{19}$$

where the asymptotic suction parameter is b_{as} = 4.

If only one component of the gas mixture is transported through the wall, then upon attainment of the limiting state in accordance with the condition $(b_1)_{lm} = \tilde{k}_{10}$ the relative Stanton number increases linearly with the penetration parameter: $\Psi = b/\tilde{k}_{10}$. This relation represents a straight line with a slope $\tan\alpha = 1/\tilde{k}_{10}$, which is independent of the flow regime. The results of calculations for various values of the concentration k_{10} of the suction component are represented by dashed lines in Fig. 3.

The behavior of the boundary layer for suction parameters greater than the limiting value is of special interest from the experimental point of view.

Figure 4 shows the relative Stanton number as a function of the suction parameter for the condensation of water vapor from vapor–gas mixtures (helium + water and Freon + water); the calculations were carried out according to expressions (17) and (18) for various values of the vapor concentration in the flow.

It follows from Fig. 4 that in the case of suction of the heavier component of a gas mixture through the wall the intensity of the transfer processes is lower, and for suction of the lighter component it is higher than in the uniform-suction case. The influence of the concentration of the condensing substance in the flow is also different; thus, for $\bar{M} > 1$ the mass-flow heat-transfer rate decreases with increasing values of $\tilde{k}_{10}$ and for $\bar{M} < 1$ the pattern is reversed.

For the condensation of vapor in a mixture with helium it is evident from Fig. 4 that the relative Stanton number prior to inception of the limiting state varies only slightly (not more than 30%) over a wide range of variation of the vapor concentration: $\tilde{k}_{10}$ = 0.1–0.9. In this case the maximum values of the penetration parameter are small, and for $\tilde{k}_{10}$ = 0.9 we have $b_{max} \approx 1.0$. The relative Stanton number also increases insignificantly for a fixed value of the suction parameter b and $\Psi < \Psi_{lm}$ in the condensation of water vapor in a mixture with Freon ($\bar{M}$ = 0.15), but now it is possible to have substantially larger suction parameters b than for $\bar{M} > 1$ with the same values of the vapor concentration in the undisturbed flow. The

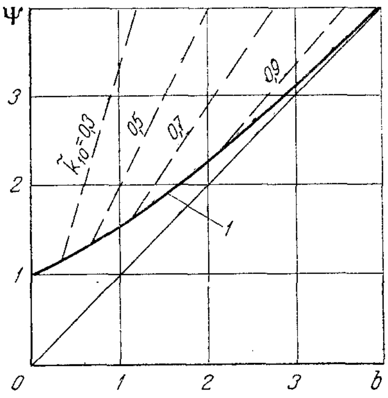

Fig. 3

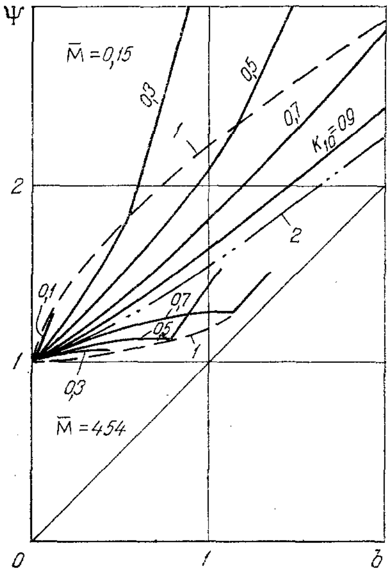

Fig. 4

Fig. 3. Relative mass-flow heat-transfer coefficient (Stanton number) for selective suction of a turbulent boundary layer, $\psi\psi_1 = 1$. 1) Calculated according to (19).

Fig. 4. Relative Stanton number for the condensation of water vapor in a mixture with Freon-12 ($\bar{M}$ = 0.15) and with helium ($\bar{M}$ = 4.54). 1) Limiting curve $\Psi = \Psi_{lm}$; 2) isothermal uniform suction, calculated according to (19).

nonuniformity parameter $\bar{M}$ for the data in Fig. 4 differs by more than an order of magnitude, whereas the relative Stanton number varies insignificantly, particularly for small suction parameters ($b < 0.6$). This fact is quite consistent with the great many experimental data [3, 4, 6-8] obtained for the condensation of various vapor—gas mixtures.

The dashed curves in Fig. 4 correspond to the limiting state of the boundary layer $b_{1d} = \tilde{k}_{10}$. As was shown above, the limiting state of the boundary layer can, by analogy with the asymptotic layer, attain both the laminar and the turbulent flow regime.

In Fig. 5 the results of the calculations are compared with the experimental data on heat and mass transfer for the condensation of water vapor in a mixture with Freon ($\bar{M}$ = 0.15) [3], of benzene C_6H_6 with air ($\bar{M}$ = 2.6), and of carbon tetrachloride CCl_4 also with air ($\bar{M}$ = 5.3) [7]. The experimental and calculated data have been processed in the form of the variable $\Psi_x = St/St_0$ (for Re_x = idem) as a function of the penetration parameter b_1.

The absence of any data on the vapor concentration in the flow core in all the published experimental studies of the condensation of vapor—gas mixtures, i.e., on the quantity that largely determines the intensity of the transfer processes, prohibits an adequately correct comparison of the experimental results. In Fig. 5, therefore, we have plotted families of analytical curves for various values of the vapor concentration in the flow core; the vertical analytical lines in these figures correspond to the condition $b_{1lm} = \tilde{k}_{10}$. It is evident from the graphs that over a wide range of variation of the nonuniformity parameter $\bar{M}$ the experimental and calculated data are in qualitative agreement.

However, a more detailed analysis of the reliability of the computational procedure requires, first, data on the concentrations of the components in the flow and, second, experimental results covering very large penetration parameters. This information would make it possible to explain the impossibility of attaining the asymptotic flow regime in a boundary layer with selective suction.

To calculate the intensity of vapor condensation on a wall for a given heat flux at the wall, i.e., the inverse problem, does not present any fundamental difficulty. It is necessary in this case to solve the integral diffusion relation (1) and the energy equation using relations (17) and (18) for the relative Stanton numbers as well as the corresponding expressions for the penetration parameters (7) and the wall concentration of the components (5).

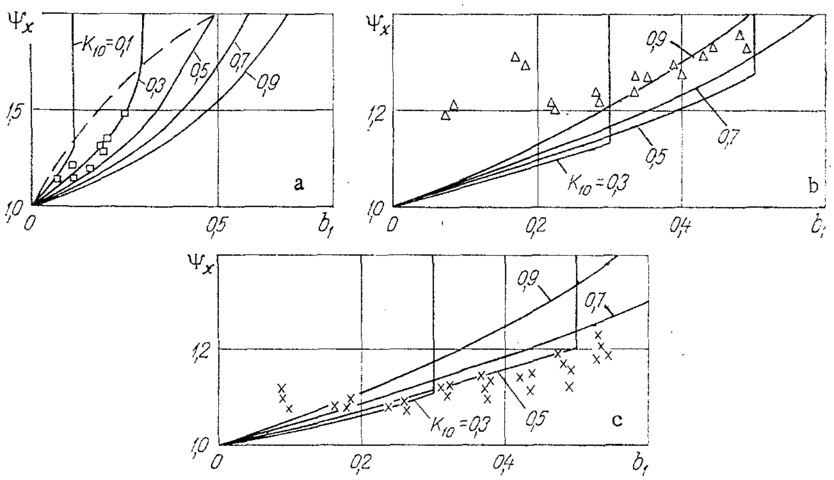

Fig. 5. Comparison of experimental and calculated data on heat and mass transfer for the condensation of vapor–gas mixtures. a) Water vapor + Freon, $\bar{M}$ = 0.15 [3]; b) benzene C_6H_6 + air, $\bar{M}$ = 2.6 [7]; c) carbon tetrachloride CCl_4 + air, $\bar{M}$ = 5.3 [7].

In conclusion, the authors gratefully acknowledge Academician S. S. Kutateladze for a valuable discussion.

NOTATION

x, y, longitudinal and transverse coordinates; $\bar{x} = x/L$, relative length; $Re_L = \rho_0 W_0 L/\mu$, $Re_d^{**} = \rho_0 w_0 \delta_d^{**}/\mu$, Reynolds numbers formed with respect to the characteristic length and the mass-reduction thickness; $\tilde{k}_i$, weight concentration; $K = r/\Delta h$, phase-transition criterion; $St = -\rho P(\partial \tilde{k}_i/\partial y)_w / \rho_0 w_0 \Delta \tilde{k}_i$, mass-flow heat-transfer coefficient (Stanton number); St_0, Stanton number under standard conditions; $\Psi = St/St_0$, relative mass-flow heat-transfer function (relative Stanton number); ρ, density; D, diffusion coefficient; $b_1 = j_w/\rho_0 w_0 St$, $b = j_w/\rho_0 w_0 St_0$, penetration (suction) parameters; ω, dimensionless velocity; $\tilde{j} = j/j_w$, quantity characterizing mass-flow profile; M, molecular weight; μ, viscosity; $\psi = T_w/T_0$, nonisothermicity factor. Indices: i, suction component; 2, uncondensed component; 0, flow core; w, wall; as, asymptotic; lm, limiting state.

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